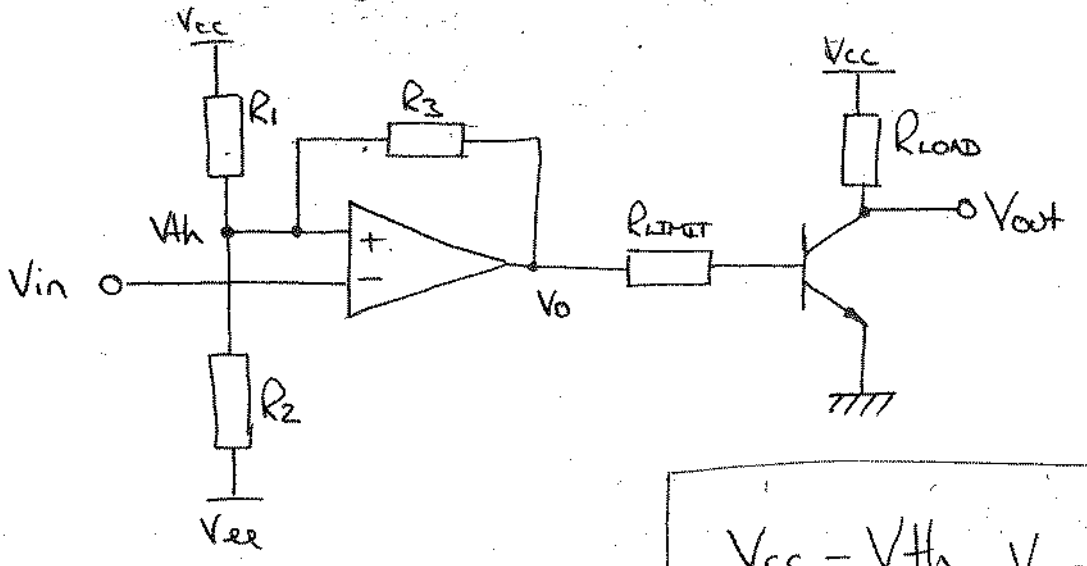
Schmitt Trigger



$V = IR$

$\frac{V}{R} = I \quad \frac{V}{I} = R$

Using Kirchoffs current law;

$I_{R1} + I_{R3} - I_{R2} = 0$

Using Ohm's law;

$I_{R1} = \frac{V_{cc} - V_{th}}{R_1}$

$I_{R2} = \frac{V_{th} - V_{ee}}{R_2}$

$I_{R3} = \frac{V_o - V_{th}}{R_3}$

$$\frac{V_{cc} - V_{th}}{R_1} + \frac{V_o - V_{th}}{R_3} - \frac{V_{th} - V_{ee}}{R_2} = 0$$

$$= \frac{R_2 R_3 (V_{cc} - V_{th})}{R_1 R_2 R_3} + \frac{R_1 R_2 (V_o - V_{th})}{R_1 R_2 R_3} - \frac{R_1 R_3 (V_{th} - V_{ee})}{R_1 R_2 R_3}$$

$$= \frac{R_2 R_3 V_{cc} - R_2 R_3 V_{th} + R_1 R_2 V_o - R_1 R_2 V_{th} - R_1 R_3 V_{th} + R_1 R_3 V_{ee}}{R_1 R_2 R_3}$$

$$= \frac{R_2 R_3 V_{cc} + R_1 R_2 V_o + R_1 R_3 V_{ee}}{R_1 R_2 R_3} + \frac{-R_2 R_3 V_{th} - R_1 R_2 V_{th} - R_1 R_3 V_{th}}{R_1 R_2 R_3}$$

$$R_2 R_3 V_{cc} + R_1 R_2 V_o + R_1 R_3 V_{ee} = -(-R_2 R_3 V_{th} - R_1 R_2 V_{th} - R_1 R_3 V_{th})$$

$$R_2 R_3 V_{cc} + R_1 R_2 V_o + R_1 R_3 V_{ee} = V_{th}(R_1 R_2 + R_2 R_3 + R_1 R_3)$$

$$V_{th} = \frac{R_2 R_3 V_{cc} + R_1 R_2 V_o + R_1 R_3 V_{ee}}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$V_{th} = \frac{R_2 R_3 V_{cc} + R_1 R_2 V_o + R_1 R_3 V_{ee}}{R_1 (R_2 + R_3) + R_2 R_3}$$

Now, we'd like to find the margin between thresholds;

$$V_{marg} = V_{th(V_{OH})} - V_{th(V_{OL})}$$

$$V_{th} = \frac{R_2R_3V_{cc} + R_1R_2V_O + R_1R_3V_{ee}}{R_1(R_2+R_3)+R_2R_3}$$

$$V_{marg} = \frac{A_1}{B} - \frac{A_2}{B}$$

$$= \frac{A_1 - A_2}{B}$$

$$A_1 - A_2 = R_2R_3V_{cc} + R_1R_2V_{OH} + R_1R_3V_{ee} - (R_2R_3V_{cc} + R_1R_2V_{OL} + R_1R_3V_{ee})$$

$$= R_1R_2V_{OH} - R_1R_2V_{OL}$$

$$= R_1R_2(V_{OH} - V_{OL})$$

$$V_{marg} = \frac{R_1R_2(V_{OH} - V_{OL})}{R_1(R_2+R_3)+R_2R_3} \Longleftarrow$$

Now we've found the margin between the thresholds, we'd like to find the centre of the space between the thresholds. This could be considered to be the "virtual threshold"

$$V_{mid} = \frac{V_{marg}}{2} + V_{th(V_{OL})} = \frac{R_1R_2(V_{OH} - V_{OL})}{2R_1(R_2+R_3)+2R_2R_3} + \frac{R_2R_3V_{cc} + R_1R_2V_{OL} + R_1R_3V_{ee}}{R_1(R_2+R_3)+R_2R_3}$$

all we need to do is simplify...

$$V_{mid} = \frac{(R_1(R_2+R_3)+R_2R_3)(R_1R_2(V_{OH}-V_{OL}))+(2R_1(R_2+R_3)+2R_2R_3)(R_2R_3V_{CC}+R_1R_2V_{OL}+R_1R_3V_{EE})}{(2R_1(R_2+R_3)+2R_2R_3)(R_1(R_2+R_3)+R_2R_3)}$$

$$V_{mid} = \frac{A+B}{C}$$

$$A = R_1R_2(V_{OH}-V_{OL}).R_1(R_2+R_3) + R_1R_2(V_{OH}-V_{OL}).R_2R_3$$
$$= (R_1R_2V_{OH} - R_1R_2V_{OL})(R_1R_2+R_1R_3) + (R_1R_2V_{OH} - R_1R_2V_{OL})R_2R_3$$
$$= R_1^2R_2^2V_{OH} + R_1^2R_2R_3V_{OH} - R_1^2R_2^2V_{OL} - R_1^2R_2R_3V_{OL} + R_1R_2^2R_3V_{OH} - R_1R_2^2R_3V_{OL}$$

$$B = (2R_1R_2+2R_1R_3+2R_2R_3)(R_2R_3V_{CC}+R_1R_2V_{OL}+R_1R_3V_{EE})$$
$$= 2R_1R_2^2R_3V_{CC} + 2R_1^2R_2^2V_{OL} + 2R_1^2R_2R_3V_{EE}$$
$$+ 2R_1R_2R_3^2V_{CC} + 2R_1^2R_2R_3V_{OL} + 2R_1^2R_3^2V_{EE}$$
$$+ 2R_2^2R_3^2V_{CC} + 2R_1R_2^2R_3V_{OL} + 2R_1R_2R_3^2V_{EE}$$

$$C = (2R_1R_2+2R_1R_3+2R_2R_3)(R_1R_2+R_1R_3+R_2R_3)$$

$$A = R_1^2R_2^2(V_{OH}-V_{OL}) + R_1^2R_2R_3(V_{OH}-V_{OL}) + R_1R_2^2R_3(V_{OH}-V_{OL})$$
$$= V_{OH}(R_1^2R_2^2+R_1^2R_2R_3+R_1R_2^2R_3) - V_{OL}(R_1^2R_2^2+R_1^2R_2R_3+R_1R_2^2R_3)$$

$$B = 2V_{OL}(R_1^2R_2^2+R_1^2R_2R_3+R_1R_2^2R_3) + ((2R_1R_2+2R_1R_3+2R_2R_3)(R_2R_3V_{CC}+R_1R_3V_{EE}))$$

$$A+B = V_{OH}+V_{OL}(R_1^2R_2^2+R_1^2R_2R_3+R_1R_2^2R_3) + ((2R_1R_2+2R_1R_3+2R_2R_3)(R_2R_3V_{CC}+R_1R_3V_{EE}))$$
$$= (2R_1R_2+2R_1R_3+2R_2R_3)(R_2R_3V_{CC}+R_1R_2(V_{OH}+V_{OL})+R_1R_3V_{EE})$$

$$\frac{A+B}{C} = \frac{(2R_1R_2+2R_1R_3+2R_2R_3)(R_2R_3V_{CC}+R_1R_2(V_{OH}+V_{OL})+R_1R_3V_{EE})}{(2R_1R_2+2R_1R_3+2R_2R_3)(R_1R_2+R_1R_3+R_2R_3)}$$

$$= \frac{R_2R_3V_{CC}+R_1R_2(V_{OH}+V_{OL})+R_1R_3V_{EE}}{R_1R_2+R_1R_3+R_2R_3}$$

$$V_{mid} = \frac{A+B}{C}$$

$$V_{mid} = \frac{R_2R_3V_{CC}+R_1R_2(V_{OH}+V_{OL})+R_1R_3V_{EE}}{R_1(R_2+R_3)+R_2R_3.} \Leftarrow$$

Notably V_{mid} & V_{th} are very similar functions It was a lot of work to get V_{mid}, perhaps a better mathematician would have seen that now seems obvious!